NAG Toolbox for MATLAB

f08fq

1 Purpose

f08fq computes all the eigenvalues and, optionally, all the eigenvectors of a complex Hermitian matrix. If the eigenvectors are requested, then it uses a divide-and-conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal–Walker–Kahan variant of the *QL* or *QR* algorithm.

2 Syntax

```
[a, w, info] = f08fq(job, uplo, a, 'n', n)
```

3 Description

f08fq computes all the eigenvalues and, optionally, all the eigenvectors of a complex Hermitian matrix A. In other words, it can compute the spectral factorization of A as

$$A = Z\Lambda Z^{\mathrm{H}},$$

where Λ is a real diagonal matrix whose diagonal elements are the eigenvalues λ_i, and Z is the (complex) unitary matrix whose columns are the eigenvectors z_i. Thus

$$Az_i = \lambda_i z_i, \qquad i = 1, 2, \ldots, n.$$

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **job – string**

Indicates whether eigenvectors are computed.

job = 'N'

Only eigenvalues are computed.

job = 'V'

Eigenvalues and eigenvectors are computed.

Constraint: **job** = 'N' or 'V'.

2: **uplo – string**

Indicates whether the upper or lower triangular part of A is stored.

uplo = 'U'

The upper triangular part of A is stored.

uplo = 'L'

The lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **a(lda,∗) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n Hermitian matrix A.

If **uplo** = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork, rwork, lrwork, iwork, liwork

5.4 Output Parameters

1: **a(lda,∗) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **job** = 'V', **a** contains the unitary matrix Z which contains the eigenvectors of A.

2: **w(∗) – double array**

Note: the dimension of the array **w** must be at least $\max(1, \mathbf{n})$.

The eigenvalues of the matrix A in ascending order.

3: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **job**, 2: **uplo**, 3: **n**, 4: **a**, 5: **lda**, 6: **w**, 7: **work**, 8: **lwork**, 9: **rwork**, 10: **lrwork**, 11: **iwork**, 12: **liwork**, 13: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

if **info** $= i$ and **job** $=$ 'N', the algorithm failed to converge; i elements of an intermediate tridiagonal form did not converge to zero; if **info** $= i$ and **job** $=$ 'V', then the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and column $i/(\mathbf{n}+1)$ through $\mathrm{mod}(i, \mathbf{n}+1)$.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A+E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* 1999 for further details.

8 Further Comments

The real analogue of this function is f08fc.

9 Example

```
job = 'V';
uplo = 'L';
a = [complex(1, +0), complex(0, 0), complex(0, 0), complex(0, 0);
     complex(2, +1), complex(2, +0), complex(0, 0), complex(0, 0);
     complex(3, +1), complex(3, +2), complex(3, +0), complex(0, +0);
     complex(4, +1), complex(4, +2), complex(4, +3), complex(4, +0)];
[aOut, w, info] = f08fq(job, uplo, a)
```

```
aOut =
  -0.4836           -0.6470           -0.4456           -0.3859
   -0.2912 + 0.3618i    0.4984 + 0.1130i   -0.0230 - 0.5702i   -0.4441 +
0.0156i
    0.3163 + 0.3696i   -0.2949 - 0.3165i    0.5331 + 0.1317i   -0.5173 -
0.0844i
    0.4447 - 0.3406i    0.2241 + 0.2878i   -0.3510 + 0.2261i   -0.5277 -
0.3168i
w =
   -4.2443
   -0.6886
    1.1412
   13.7916
info =
           0
```